20 March 1995

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 199 (1995) 65–69

Controlling chaos in discrete neural networks

Ricard V. Solé, Liset Menéndez de la Prida

Complex Systems Research Group, Departament de Fisica i Enginyeria Nuclear, Universitat Politécnica de Catalunya, Sor Eulàlia d'Anzizu s/n, Campus Nord, Mòdul B4, 08034 Barcelona, Spain

Received 11 November 1994; accepted for publication 11 January 1995
Communicated by A.P. Fordy

Abstract

A recently developed method for chaos control [Phys. Lett. A 181 (1993) 29] is applied to small discrete neural networks. This method is based on the application of periodic proportional pulses. In our study, this control is applied to one of the neurons, the others being free of direct control. It is shown that control of unstable periodic orbits is reached for a wide set of parameters.

Neural networks have been widely used as models of real neural structures from small networks of neurons to large scale neurodynamics [1,2]. Complex spatial and temporal patterns are observed both in real and simulated systems. Among them chaotic behaviour is often involved, and low-dimensional strange attractors have been observed in brain dynamics [3] as well as in small sets of coupled neurons [2]. The existence of chaos in brain dynamics is probably related with the underlying mechanism involved in neural computation, but many open questions still remain, as for example how to prove that true deterministic chaos is present in neural structures.

Recently, a new possibility has emerged from the theory of chaos control (see Ref. [4] and references therein). A standard method of controlling chaos is the well known OGY method [4], where unstable periodic orbits are stabilized by means of small external perturbations to system parameters. Among the spectrum of applications of chaos control, neural systems are a particularly interesting class of complex structures where chaos control has been shown to apply [5]. The consequences of this stabilization for neural systems theory are twofold. First, because chaos control can only be obtained if deterministic chaos is involved, and second, because of its implications for neural computation. Practical applications to the treatment of epileptic foci have been recently conjectured [6].

A new method for chaos control has been recently proposed by Güemez and Matías [7,8] and is based on the application of periodic, proportional feedbacks acting on the system variables (instead of system parameters). The GM method was applied to one-dimensional maps [7], i.e. maps $x_{n+1} = F_\mu(x_n)$, as follows,

$$x_{n+1} = F_\mu(x_n)(1 + \gamma\delta_{n,p}), \tag{1}$$

where $\delta_{n,p} = 1$ when n is a multiple of $p \in \mathbb{N}$ and zero otherwise. So we can see that every p time steps our system is modified by means of a proportional feedback of strength $\gamma \in \mathbb{R}$. Using this method, we hope to stabilize unstable periodic orbits of periodicity equal to a multiple of p. The simplicity of this method makes possible a straightforward application to neural

SSDI 0375-9601(95)00040-2

systems. In this Letter, we consider a particular case of chaos control on neural networks based upon the GM method.

Let us consider a general situation involving discrete dynamical systems, defined as an m-dimensional map $\boldsymbol{X}_{n+1} = \boldsymbol{F}_{\mu}(\boldsymbol{X}_n)$, where $\boldsymbol{X} \in \mathbb{R}^m$. Specifically, we deal with an m-neuron fully connected network, defined as

$$x^i_{n+1} = \Phi_\mu\left(\sum_{j=1}^{m} W_{ij} x^j_n\right),$$

with $i = 1, \dots, m$. $\Phi_\mu(z)$ is assumed to be a sigmoidal function and we take $\Phi_\mu(z) = (1 + e^{-\mu z})^{-1}$.

In this Letter, the GM method is applied on a single neuron of the m-network, i.e. the new set of equations is

$$x^i_{n+1} = \Phi_\mu\left(\sum_{j=1}^{m} W_{ij} x^j_n\right)(1 + \gamma\delta_{n,p}), \tag{2a}$$

$$x^k_{n+1} = \Phi_\mu\left(\sum_{j=1}^{m} W_{ij} x^j_n\right),$$

$$1 \leqslant k \leqslant m \quad (k \neq i), \tag{2b}$$

where, as we can see, only the ith neuron is controlled.

To begin with, let us consider the $m = 2$ case, i.e. a 2D fully connected neural network defined as

$$x_{n+1} = \Phi_\mu(W_{11}x_n + W_{12}y_n), \tag{3a}$$

$$y_{n+1} = \Phi_\mu(W_{21}x_n + W_{22}y_n). \tag{3b}$$

Because it depends on the connectivity matrix $\mathbf{W} = (W_{ij})$, this map can generate complex dynamical patterns, including deterministic chaos. We start our study with a 2D neural network with matrix

$$\mathbf{W} = \begin{pmatrix} -a & a \\ -b & b \end{pmatrix}. \tag{4}$$

For this particular choice, it has been shown [9] that this simple neural network is dynamically equivalent to a one-parameter family of S-unimodal maps of the interval [0, 1], which is well known to become chaotic via the Feigenbaum scenario. In Figs. 1a–1c we show the bifurcation scenario (a) for increasing μ-values for $a = 5, b = 25$, as well as the largest Lyapunov exponent (b) for the fully connected network shown in (c).

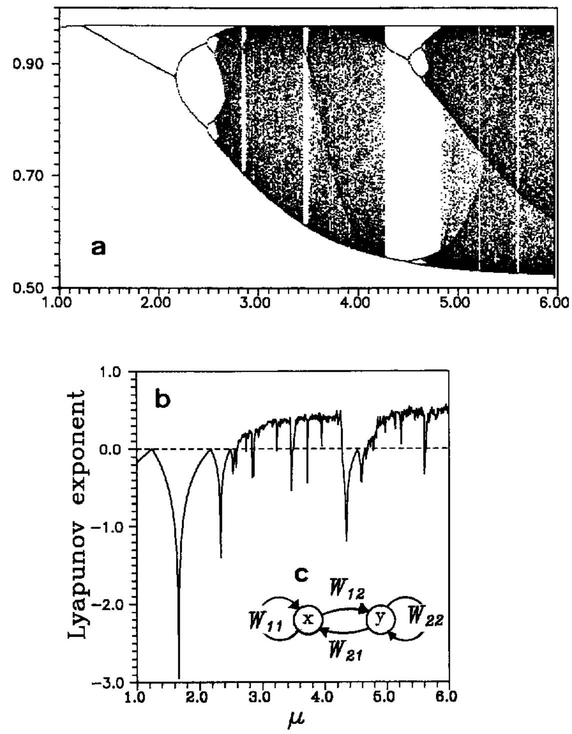

Fig. 1. (a) Bifurcation diagram and (b) largest Lyapunov exponents for the $m = 2$ neural network, defined in (3a), (3b) and with connectivity matrix $\mathbf{W}$ given in (4). In (c) a schematic representation of the network is given.

For $m = 2$ (now $x^1 = x$ and $x^2 = y$) and for $\mathbf{W}$ given by (4) we have used the GM method acting on the x-neuron. The strength γ was taken in a small interval around $\gamma \in [-\gamma_0, \gamma_0]$ of small value. In Figs. 2a, 2b we see two examples of stabilization of unstable periodic orbits by means of the GM method. Here we have (a) $\gamma = -0.01$, $p = 2$ and $\mu = 5$; for (b) we have $\gamma = -0.02$, $p = 3$ and $\mu = 6$. In both cases stabilization was obtained for the desired two- and three-periodic orbits.

In order to understand how the GM method works on our system, let us consider an example. We use $\mu = 5$ and test the GM method for an unstable orbit of period two. In Fig. 3 we show the attractor (confined to the square $[0.5, 1] \times [0.5, 1]$) as well as the two points (a) $P^* = (x^*_1, y^*_1)$ and (b) $Q^* = (x^*_2, y^*_2)$ belonging to the unstable orbit. The stable and unstable local manifolds are indicated by means of the corresponding eigenvectors. The sigmoidal character of the $\Phi_\mu(z)$

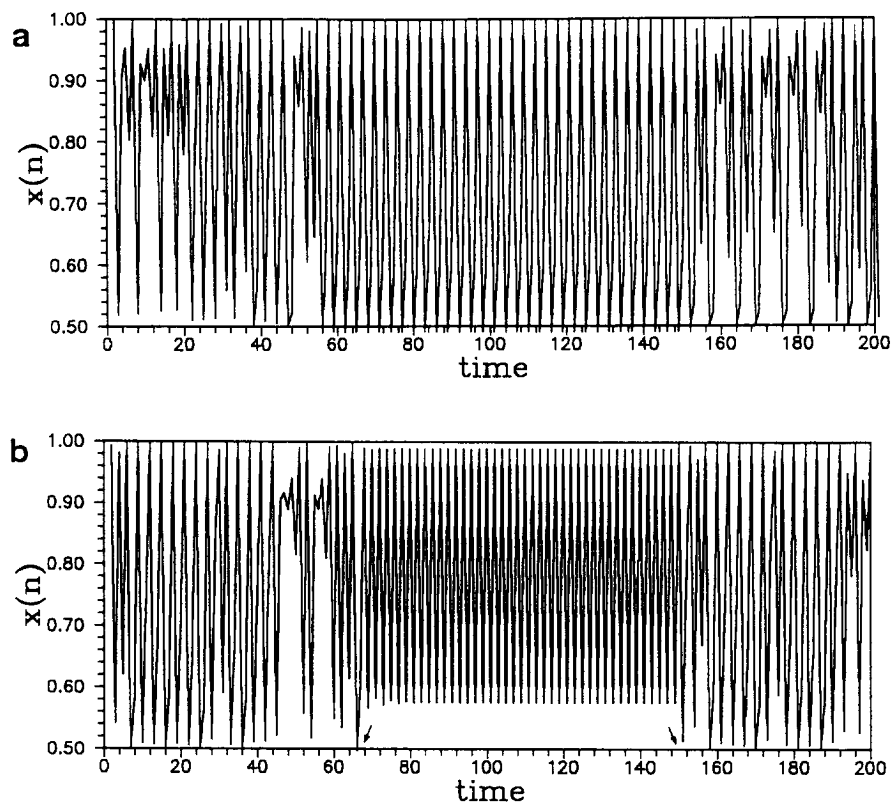

Fig. 2. Examples of chaos control for the same system as in Fig. 1. Now (a) $\mu = 5, \gamma = -0.01$ and $p = 2$; (b) $\mu = 6, \gamma = -0.02$ and $p = 3$. Control starts at $n = 70$ and is removed at $n = 150$. Two- and three-periodic orbits are stabilized.

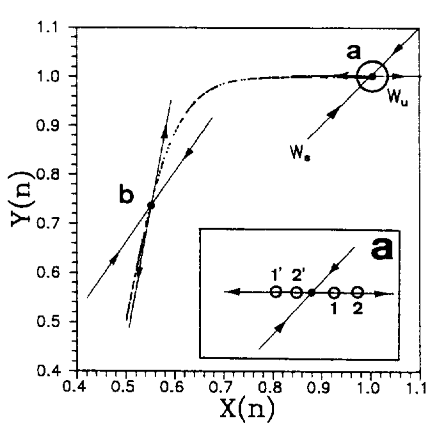

Fig. 3. Stabilization of a $2P$ orbit. The periodic unstable orbit (P^*, Q^*) is shown, together with the local stable and unstable manifolds (see text). Bottom right: a close ϵ-neighborhood of P^* is shown. We see that points close to the left-hand side $(1,2)$ are shifted to the left-hand side close to P^* (here $(1',2')$).

function makes the visits of the chaotic orbit to any given neighborhood of P^* very frequent. For example, we take the interval $B(\epsilon) = (x_1^* - \epsilon, x_1^* + \epsilon)$ around P^*, with $\epsilon = 0.001$, it can be shown that the chaotic orbit enters $B(\epsilon)$ as much as 60% of the time. This situation seems to be a fairly general one for these types of networks. Numerical simulations shows that stabilization takes place if γ is chosen such that when the trajectory enters the right-hand side of $B(\epsilon)$, the control term drives it to the left-hand side, closer to the unstable point P^*. This situation is repeated in other iterations, and the periodic orbit is stabilized.

The period p of feedback control gives us the expected periodicity, and in our study we have observed that for high values of the gain parameter μ (typically $\mu \geqslant 7$) the p-periodic stabilized orbits shows a period-doubling scenario as $|\gamma|$ becomes smaller (Fig. 4a). When $|\gamma|$ is too small, chaos control is obtained only at small periodic windows. As we can see, when $\gamma > 0$, the orbit is shifted towards the other basin of attraction (here $[0, 0.5] \times [0, 0.5]$).

The transitions from different periodicities can be measured by means of the normalized Boltzmann entropy, defined as

$$S(M) = -\frac{1}{\log M} \sum_{k=1}^{M} P(B_k) \log[P(B_k)], \qquad (5)$$

where we have divided the interval of variation of the x-variable into M sub-intervals B_k of equal length. Then we measure the number of points N_k of a given orbit of length N which falls into B_k, i.e. $P(B_k) = N_k/N$, as usual. For a given p-periodic orbit, and for a fine enough partition, we expect an entropy value of $S(p) = \log p$ and so $0 \leqslant S(p) = S(M) \leqslant 1$. If a period-doubling scenario is involved, we expect a set of step-like valued increasing entropies linked with higher-order periodicities, as shown in Fig. 4b.

Finally, we have analysed several neural networks with a larger number of neurons ($m > 2$) and control has also been achieved in spite of the higher dimensionality of the maps. In this sense fractal dimensions were computed in order to be sure that the strange attractor was not nearly one-dimensional. In Figs. 5a, 5b we show two typical examples of stabilized orbits obtained from (a) $m = 3$ and (b) $m = 4$ networks with $\mu = 4$ and $p = 2$. In the first case a four-periodic orbit was stabilized ($\gamma = -0.1$) and a two-periodic orbit in the second case ($\gamma = -0.7$). These results were obtained with the following connectivity matrices,

$$\mathbf{W}(m=3) = \begin{pmatrix} -a & a & -c \\ -b & b & -c \\ -c & c & -c \end{pmatrix}, \qquad (6)$$

$$\mathbf{W}(m=4) = \begin{pmatrix} -a & a & -c & d \\ -b & b & -c & d \\ -b & b & -c & d \\ -d & d & d & d \end{pmatrix}, \qquad (7)$$

where $c = 2$, $d = 0.1$ and $\{a, b\}$ as before.

Again, the control of a single neuron enables us to control the global dynamics of the network. Here a systematic exploration of parameters is difficult because of the intrinsic dimensionality of parameter space ($m^2 + 1$). We hope to report a general study of these higher dimensional cases in a future work as well as the extension to continuous neural networks [10]. It is our belief that these results can be useful in future studies of chaos control in neurodynamics.

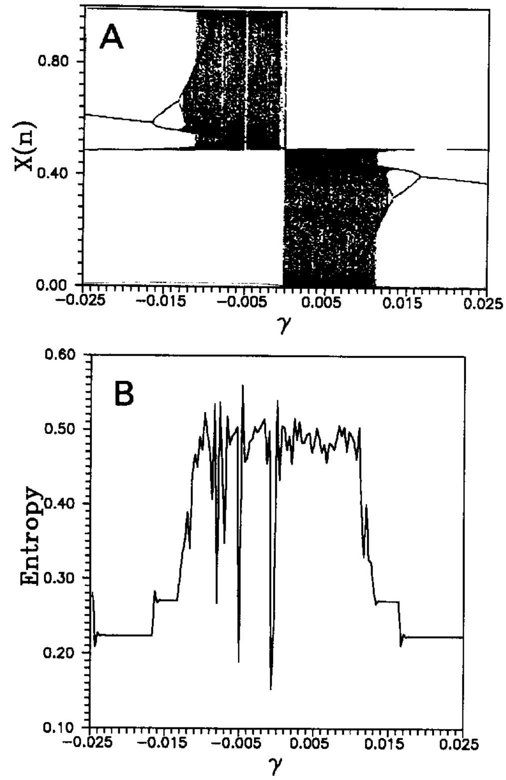

Fig. 4. (a) Bifurcation diagram of the 2D neural network for $\mu = 7$ and $p = 3$ (other parameters than in Fig. 1) for different γ-values. (b) Normalized entropy calculated from (5) using $M = 50$ and 2000 time steps after transients are removed.

The authors would like to thank Susanna C. Manrubia for several comments and discussions.

References

[1] D. Amit, Modelling brain function (Cambridge Univ. Press, Cambridge, 1989).
[2] E. Basar, H. Flohr and H. Haken, eds., Synergetics of the brain (Springer, Berlin, 1983).
[3] E. Basar, Chaos in brain function (Springer, Berlin, 1990).
[4] T. Shinbrot, C. Grebogi, E. Ott and J.A. Yorke, Nature 363 (1993) 411.
[5] S.J. Schiff, K. Jerger, D.H. Duong, T. Chang, M.L. Spanno and W.L. Ditto, Nature 370 (1994) 615;
J.A. Sepulchre and A. Babloyantz, Phys. Rev. E 48 (1993) 945.
[6] J. Glanz, Science 265 (1994) 174.

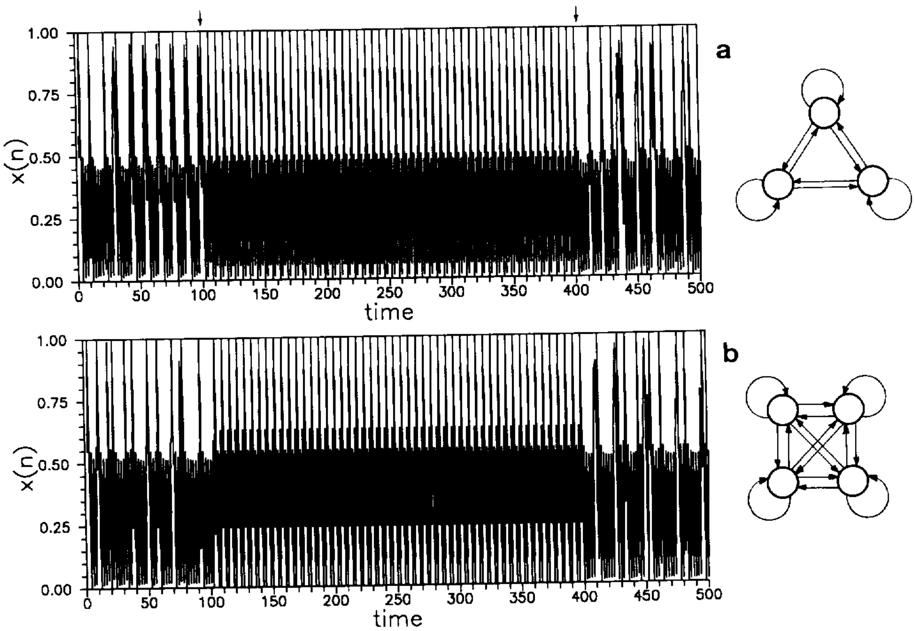

Fig. 5. Chaos control for (a) three- and (b) four-dimensional neural networks. The connectivity matrices **W** are given in (6) and (7). Here (a) $\mu = 4$, $p = 2$ and $\gamma = -0.1$ and (b) $\mu = 4$, $p = 2$ and $\gamma = -0.7$ control starts at $n = 100$ and is removed at $t = 400$.

[7] J. Güemez and M.A. Matías, Phys. Lett. A 181 (1993) 29.
[8] M.A. Matías and J. Güemez, Phys. Rev. Lett. 72 (1994) 1455.
[9] X. Wang, Complex Systems 5 (1991) 425.
[10] L. Menéndez de la Prida and R.V. Solé, in preparation.